Math 10B with Professor Stankova
Worksheet, Discussion #11; Thursday, 2/28/2019
GSI name: Roy Zhao

1 Independence

1.1 Concepts

1. We say that two events A, B are **independent** if $P(A \cap B) = P(A)P(B)$.

1.2 Examples

2. When rolling a fair 6-sided die, are the events A that the number rolled is greater than or equal to 3, and B that the number rolled is odd, independent?

> **Solution:** We just need to check $P(A \cap B) = P(A)P(B)$. On the left side, the probability is $\frac{2}{6}$ from having $3, 5$, and $P(A) = \frac{2}{3}$ and $P(B) = \frac{1}{2}$ so indeed $P(A \cap B) = P(A)P(B)$. So they are independent.

1.3 Problems

3. **TRUE** False If A, B are mutual exclusive events that are independent, then $P(A) = 0$ or $P(B) = 0$.

> **Solution:** If A, B are mutually exclusive, then $A \cap B = \emptyset$. Then if they are independent, then $P(A \cap B) = 0 = P(A)P(B)$ so $P(A) = 0$ or $P(B) = 0$.

4. True **FALSE** If A, B are independent events and B, C are independent, then A, C are independent.

> **Solution:** We can take A and C to be the same event.

5. I roll two die. Are the events that the first die roll is a 1 and that the sum of the two dice is a 7 independent?

Solution: Let A be the event that the first die roll is a 1 and let B be the event that the sum of the two dice is a 7. Then we can compute $P(A) = P(B) = 1/6$ and $P(A \cap B) = 1/36$, so they are independent.

6. Let E be the event that a randomly generated bit string of length three contains an odd number of 1s and let F be the event that the string starts with 1. Are E and F independent?

Solution: $P(E) = \frac{\binom{3}{1}+\binom{3}{3}}{2^3} = \frac{4}{8} = \frac{1}{2}$ and $P(F) = \frac{1}{2}$ while $P(E \cap F) = \frac{2}{8} = \frac{1}{4} = P(E)P(F)$ so E and F are independent.